LETTER TO THE EDITOR

Two-mode squeezed vacuum state coupled to the common thermal reservoir

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Abstract
Entangled states play a crucial role in quantum information protocols, thus the dynamical behaviour of entanglement is of great importance. In this letter, we consider a two-mode squeezed vacuum state coupled to one thermal reservoir as a model of an entangled state embedded in an environment. As a criterion for entanglement we use a continuous-variable equivalent of the Peres–Horodecki criterion, namely the Simon criterion. To quantify entanglement we use the logarithmic negativity. We derive a condition, which assures that the state remains entangled in spite of the interaction with the reservoir. Moreover for the case of interaction with vacuum as an environment we show that a state of interest after infinitely long interaction is not only entangled, but also pure. For comparison we also consider a model in which each mode is coupled to its own reservoir.

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1. Introduction

A number of possible practical applications of entangled states have been proposed, including quantum computation [1] and quantum teleportation [2, 3]. Thus it is very important to know how the interaction with an environment influences the dynamical behaviour of entanglement. In many experiments the process of the parametric down conversion in nonlinear crystals [4, 5] serves as a source of entangled pairs, in which two-mode squeezed vacuum states are produced. Hence, here we consider the time evolution of the two-mode squeezed vacuum, as an example of a continuous-variable entangled state, embedded in a thermal environment. Moreover, let us assume that, for some reason, e.g. storage, both modes are captured in a resonant cavity, i.e. they are coupled to a common reservoir. Additionally we presume that they have the same frequency. There has been intensive research in this direction, but Jeong *et al* [6], Kim and Lee [7], Scheel *et al* [8], Hiroshima [9] and Paris [10] assumed that the two-mode squeezed vacuum interacts with the environment in a different way. In their

model both modes are spatially separated and thus each of them is independently coupled to its own reservoir. Both modes are taken to have the same frequency and the same coupling constant. This model is justified while the two-mode squeezed vacuum can be used as a quantum channel between sender and receiver stations in quantum teleportation [3]. There is a possible experimental investigation of degradation of entanglement due to the interaction with the environment [11].

2. Model and description

In this letter, we consider a bipartite system, namely, two modes of the electromagnetic field. Each of these modes has the Hilbert space equivalent to the Hilbert space of the harmonic oscillator. The Hilbert space of the system of interest is a tensor product of the subsystems' Hilbert spaces, $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. Each mode is described by dimensionless quadrature operators defined as $\hat{x}_j = (\hat{a}_j + \hat{a}_j^\dagger)/\sqrt{2}$ and $\hat{p}_j = i(\hat{a}_j^\dagger - \hat{a}_j)/\sqrt{2}$ ($j = 1, 2$). These operators obey a standard commutation relation $[\hat{x}_j, \hat{p}_k] = i\delta_{jk}$, which is equivalent to $[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{jk}$.

The bipartite system in question is embedded in the thermal environment, which we imitate by the set of harmonic oscillators. We model the coupling of both modes to the common reservoir by the interaction Hamiltonian of the form:

$$H_{\text{one}}^{\text{int}} = \eta \sum_{i=1,2} \left(\hat{a}_i \sum_{k=1}^\infty \hat{b}_k^\dagger + \hat{a}_i^\dagger \sum_{k=1}^\infty \hat{b}_k \right). \tag{1}$$

While in the case when each mode interacts with its own reservoir, the interaction Hamiltonian is

$$H_{\text{two}}^{\text{int}} = \eta \left(\hat{a}_1 \sum_{k=1}^\infty \hat{b}_k^\dagger + \hat{a}_1^\dagger \sum_{k=1}^\infty \hat{b}_k + \hat{a}_2 \sum_{l=1}^\infty \hat{c}_l^\dagger + \hat{a}_2^\dagger \sum_{l=1}^\infty \hat{c}_l \right). \tag{2}$$

In both cases η is a coupling constant.

In order to study the evolution of our system we use the Wigner distribution function defined as [12]:

$$W(x, p) = \pi^{-2} \int d^2 x' \langle x - x' | \hat{\rho} | x + x' \rangle \exp(2ix' \cdot p) \tag{3}$$

where $\hat{\rho}$ is the density operator, $x = (x_1, x_2)$ and $p = (p_1, p_2)$.

For the sake of convenience, we arrange the quadrature operators and the phase space variables into four-dimensional vectors $\hat{X} = (\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2)$ and $\mathbf{X} = (x_1, p_1, x_2, p_2)$, and define a *quadrature variance matrix* [13]:

$$V_{\alpha\beta} = \langle \{\Delta \hat{X}_\alpha, \Delta \hat{X}_\beta\} \rangle = \text{Tr}(\{\Delta \hat{X}_\alpha, \Delta \hat{X}_\beta\}\hat{\rho})$$
$$= \int d^4 X \Delta X_\alpha \Delta X_\beta W(\mathbf{X}) \tag{4}$$

where $\{\Delta \hat{X}_\alpha, \Delta \hat{X}_\beta\} = (\Delta \hat{X}_\alpha \Delta \hat{X}_\beta + \Delta \hat{X}_\beta \Delta \hat{X}_\alpha)/2$. Of course $\Delta \hat{X}_\alpha = \hat{X}_\alpha - \langle \hat{X}_\alpha \rangle$ and $\langle \hat{X}_\alpha \rangle = \text{Tr}(\hat{X}_\alpha \hat{\rho})$. Similarly, $\Delta X_\alpha = X_\alpha - \langle X_\alpha \rangle$, where $\langle X_\alpha \rangle$ is calculated as an average of X_α with respect to the Wigner distribution $W(\mathbf{X})$. Let us stress that $\langle X_\alpha \rangle$ equals $\langle \hat{X}_\alpha \rangle$.

A (zero-mean) Gaussian state is described by the Wigner distribution in the form:

$$W(\mathbf{X}) = (2\pi)^{-2} (\det \mathbf{V})^{-1/2} \exp\left(-\tfrac{1}{2} \mathbf{X} \mathbf{V}^{-1} \mathbf{X}^T\right) \tag{5}$$

where $\mathbf{V}$ is the variance matrix defined in equation (4).

To verify whether our system is in an entangled state we use the generalization of the Peres–Horodecki criterion [14, 15] for continuous variables, derived by Simon [16]. In terms of the quadrature variance matrix, equation (4), rewritten as follows:

$$\mathbf{V} = \begin{bmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{C}^T & \mathbf{B} \end{bmatrix} \tag{6}$$

it takes the form of an inequality [16]:

$$\det \mathbf{A} \det \mathbf{B} + \left(\tfrac{1}{4} - |\det \mathbf{C}|\right)^2 - \mathrm{Tr}[\mathbf{AJCJBJC}^T\mathbf{J}] - \tfrac{1}{4}(\det \mathbf{A} + \det \mathbf{B}) \geqslant 0 \tag{7}$$

where the matrix $\mathbf{J}$ is

$$\mathbf{J} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \tag{8}$$

Inequality (7) is a *necessary* condition on the variance matrix of a separable bipartite state [16]. Moreover, Simon [16] has shown that this is also a *sufficient* condition for separability for all bipartite Gaussian states composed of two modes, each mode held by each party[1]. Thus for Gaussian states considered in this paper, when the inequality (7) is violated the state is entangled, otherwise it is separable.

Let us note that an equivalent approach is possible to the separability properties of quantum two-party Gaussian states in the framework of the operator formalism for the density operator, as has been shown by Englert and Wódkiewicz [18].

To compute the degree of entanglement we use the *logarithmic negativity* [19]:

$$E(\rho) = \log_2 \|\hat{\rho}^{T_A}\|_1 \tag{9}$$

where $\|\hat{\rho}^{T_A}\|_1$ denotes the trace norm of $\hat{\rho}^{T_A}$, and $\hat{\rho}^{T_A}$ is a partial transpose of the bipartite state $\hat{\rho}$ with respect to the subsystem A. This measure can be regarded as a quantitative version of the Peres–Horodecki criterion [14, 15], because it quantifies the degree to which $\hat{\rho}^{T_A}$ fails to be positive [19].

For states described by the Wigner function of the form (5), computing the logarithmic negativity (9) simplifies to a process known as symplectic diagonalization of the variance matrix (4), as has been shown in [19], i.e. if (λ_1, λ_2) is the symplectic spectrum of the variance matrix (4), then the logarithmic negativity equals

$$E(\rho) = F(\lambda_1) + F(\lambda_2) \tag{10}$$

where $F(\lambda_i)$ $(i = 1, 2)$ is

$$F(\lambda_i) = \begin{cases} 0 & \text{for} \quad 2\lambda_i \geqslant 1 \\ -\log_2(2\lambda_i) & \text{for} \quad 2\lambda_i < 1. \end{cases} \tag{11}$$

Evolution of two modes of the electromagnetic field coupled to the common thermal reservoir might be described by the diffusion equation, i.e. the Fokker–Planck equation. Namely, for the interaction Hamiltonian of the form (1) it reads (in the interaction picture):

$$\partial_t W(\mathbf{X}, t) = \sum_{j,k=1,2} \left[\frac{\gamma}{2}\left(\partial_{x_j} x_k + \partial_{p_j} p_k\right) + \frac{2\bar{N}+1}{4}\left(\partial^2_{x_j,x_k} + \partial^2_{p_j,p_k}\right) \right] W(\mathbf{X}, t) \tag{12}$$

where γ is a coupling constant and $\bar{N}$ is a mean thermal photon number. Here it is worthwhile to note that in the case when one couples each mode to its own reservoir, i.e. the interaction Hamiltonian is of the form (2), the relevant Fokker–Planck equation is obtained from equation (12) by substituting $j = k$.

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[1] In fact, for a bipartite Gaussian state such that one party holds one mode and the other party holds N modes positivity of the partial transpose is still a necessary and sufficient criterion for separability [17].

3. Results

The process of parametric down conversion in nonlinear crystals is used as a source of entangled pairs in many experiments [4, 5]. In such a process two-mode squeezed vacuum states are produced. Thus as an initial state of our system we consider the two-mode squeezed vacuum, which is described by the Wigner function of the form (5) with the variance matrix as follows:

$$\mathbf{V} = \frac{1}{2} \begin{bmatrix} n_1 & 0 & c_1 & 0 \\ 0 & n_2 & 0 & c_2 \\ c_1 & 0 & n_1 & 0 \\ 0 & c_2 & 0 & n_2 \end{bmatrix} \tag{13}$$

where n_i and c_i are

$$n_i = \cosh(2r) \qquad c_2 = -c_1 = \sinh(2r) \tag{14}$$

where r is a squeezing parameter (for $r \neq 0$ this state is entangled). Without loss of generality we have assumed that the squeezing parameter, r, is real.

While our system evolves, the general form of the variance matrix, equation (13), remains constant, but elements n_i and c_i are changing. Thus we may rewrite the Simon criterion, equation (7), in the form:

$$\tfrac{1}{16}[(n_1 - |c_1|)(n_2 - |c_2|) - 1][(n_1 + |c_1|)(n_2 + |c_2|) - 1] \geqslant 0. \tag{15}$$

The Simon criterion, equation (7), is a yes–no test for entanglement and thus only the sign of the left-hand side of inequality (15) is important. Therefore, we omit an expression, which does not change the sign of inequality (15). Namely, because $|c_i|$ is always positive, unless it is equal to zero and $n_1 n_2$ is always greater than one, which is a consequence of the uncertainty principle, it is more convenient to check the sign of the following inequality, instead of using inequality (15), [7]:

$$(n_1 - |c_1|)(n_2 - |c_2|) - 1 \geqslant 0. \tag{16}$$

When it is satisfied then the state is separable, otherwise it is entangled.

Solving equation (12) with the initial variance matrix in the form of equation (13) with n_i and c_i given in equation (14) leads to the time-dependent form of n_i and c_i:

$$\begin{aligned} n_1 &= \tfrac{1}{2}[2\cosh(2r) + (N - e^{-2r})\tau] \\ n_2 &= \tfrac{1}{2}[2\cosh(2r) + (N - e^{2r})\tau] \\ c_1 &= \tfrac{1}{2}[(N - e^{-2r})\tau - 2\sinh(2r)] \\ c_2 &= \tfrac{1}{2}[(N - e^{2r})\tau + 2\sinh(2r)] \end{aligned} \tag{17}$$

where $\tau = 1 - e^{-2\gamma t}$, $N = 2\bar{N} + 1$ and γ denotes coupling. Then substituting (17) into inequality (16) we find that if the initial state is sufficiently squeezed, i.e.,

$$|r| \geqslant \tfrac{1}{2}\ln(2\bar{N} + 1) \tag{18}$$

it will remain entangled forever in spite of interaction. Otherwise, the state will disentangle after time:

$$t = \frac{1}{2\gamma}\ln\left(\frac{2\bar{N} + 1 - e^{-2|r|}}{2\bar{N} + 1 - e^{2|r|}}\right). \tag{19}$$

Thus based on condition (18) it is possible to choose such an initial state of the two-mode squeezed vacuum, for which the entanglement will survive the interaction with the common reservoir.

Inequality (18) seems to have the same form as a condition for non-classicality of a squeezed thermal state [20]. In the latter case, however, $\bar{N}$ is a mean thermal photon number and r is a squeezing parameter, both corresponding to the thermal squeezed state. Here $\bar{N}$ corresponds to a number of thermal photons in the environment and r is the squeezing parameter of a state embedded in this environment.

For comparison, if one takes into consideration the coupling to two independent thermal reservoirs, it will lead to the same general form of the quadrature variance matrix, equation (13), and the separability condition, equation (16), but coefficients n_i and c_i would be different, namely [6, 7, 21]:

$$n_i = \cosh(2r)\,e^{-\gamma t} + N(1 - e^{-\gamma t}) \qquad c_2 = -c_1 = \sinh(2r)\,e^{-\gamma t} \qquad (20)$$

where, as before, r is a squeezing parameter, $N = 2\bar{N} + 1$ and γ denotes coupling. In such a case, every state will disentangle after the time:

$$t = \frac{1}{\gamma} \ln \left(1 + \frac{1 - e^{-2|r|}}{2\bar{N}} \right) \qquad (21)$$

unless it would interact with vacuum, as has been shown by Duan *et al* [21], Scheel *et al* [8] and Paris [10]. Such a situation corresponds to the limit $\bar{N} \to 0$, which implies $t \to \infty$.

For the variance matrix of the form (13) it is easy to find the symplectic spectrum and the logarithmic negativity. The symplectic spectrum takes the form:

$$\lambda_1 = \tfrac{1}{2}\sqrt{(n_1 - c_1)(n_2 + c_2)} \qquad \lambda_2 = \tfrac{1}{2}\sqrt{(n_1 + c_1)(n_2 - c_2)}. \qquad (22)$$

Using the above-mentioned symplectic spectrum, equation (22), and conditions for function $F(\lambda_i)$, equation (11), together with equation (17) it is possible to derive inequality (18) and the expression for the disentanglement time in the one reservoir case, equation (19). Analogously it is possible to derive a formula for the disentanglement time in the case of the interaction with two reservoirs, equation (21).

In particular, the symplectic spectrum, equation (22), holds for an initial two-mode squeezed vacuum state with n_i and c_i given by equation (14), and leads to the conclusion that the logarithmic negativity for such a state is simply proportional to the absolute value of the squeezing parameter, r:

$$E(\rho) = \frac{2}{\ln 2}|r|. \qquad (23)$$

Moreover, having the time-dependent form of n_i and c_i, equation (17), allows us to write down the analytic expression for the asymptotic value of the logarithmic negativity at $t \to \infty$:

$$E(\rho) = \frac{1}{\ln 2}|r| - \frac{1}{2}\log_2(2\bar{N} + 1) \qquad (24)$$

which is valid for initial states fulfilling condition (18). In the case when $|r| = \frac{1}{2}\ln(2\bar{N} + 1)$, $\lim_{t\to\infty} E(\rho) = 0$ (compare with the event of the interaction of two-mode squeezed state with two separate reservoirs in a vacuum state).

Figures 1 and 2 show changes of the logarithmic negativity, $E(\rho)$, during the interaction with the common reservoir for different initial values of the squeezing parameter, r, and the mean thermal photon number, $\bar{N}$. Logarithmic negativity, $E(\rho)$, is plotted as a function of the dimensionless rescaled time $\tau = 1 - \exp(-2\gamma t)$ which equals 0 for $t = 0$ and 1 for $t = \infty$. Note that the coupling constant, γ, changes only the time scale and has no effect on the logarithmic negativity itself. From figure 1 it is easy to note that states satisfying condition (18) (i.e. for $r = 0.5$, $r = 1$ and $r = 2$) remain entangled forever in spite of interaction.

Figures 3 and 4 show changes of the logarithmic negativity, $E(\rho)$, during the interaction with two separated reservoirs for different values of the squeezing parameter, r, and the mean
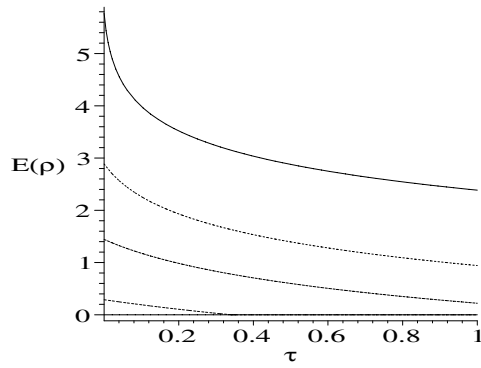
Figure 1. Logarithmic negativity, $E(\rho)$, for the two-mode squeezed state as a function of dimensionless time, $\tau = 1 - \exp(-2\gamma t)$, which is 0 for $t = 0$ and 1 for $t = \infty$, in the common reservoir model, for $\bar{N} = 0.5$ and $r = (0, 0.1, 0.5, 1, 2)$ (from the bottom to the top).
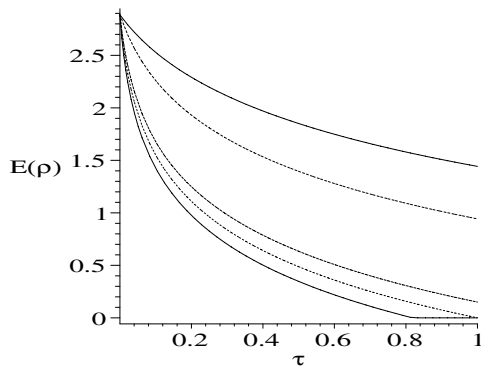

Figure 2. Logarithmic negativity, $E(\rho)$, for the two-mode squeezed state as a function of dimensionless time, $\tau = 1 - \exp(-2\gamma t)$, which is 0 for $t = 0$ and 1 for $t = \infty$, in the common reservoir model, for $r = 1$ and $N = (1, 2, 6, \exp(2), 9)$ (from the top to the bottom), $N = 2\bar{N} + 1$.
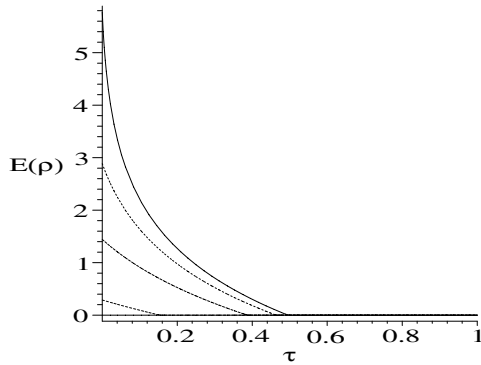

Figure 3. Logarithmic negativity, $E(\rho)$, for the two-mode squeezed state as a function of dimensionless time, $\tau = 1 - \exp(-\gamma t)$, which is 0 for $t = 0$ and 1 for $t = \infty$, in the two reservoir model, for $\bar{N} = 0.5$ and $r = (0, 0.1, 0.5, 1, 2)$ (from the bottom to the top).

thermal photon number, $\bar{N}$. Again the logarithmic negativity, $E(\rho)$, is plotted as a function of the dimensionless time $\tau = 1 - \exp(-\gamma t)$ which equals 0 for $t = 0$ and 1 for $t = \infty$. From
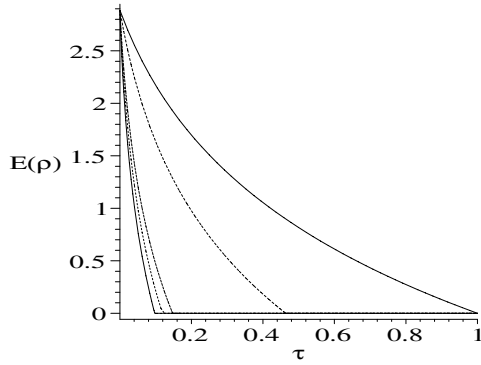
Figure 4. Logarithmic negativity, $E(\rho)$, for the two-mode squeezed state as a function of dimensionless time, $\tau = 1 - \exp(-\gamma t)$, which is 0 for $t = 0$ and 1 for $t = \infty$, in the two reservoir model, for $r = 1$ and $N = (1, 2, 6, \exp(2), 9)$ (from the top to the bottom), $N = 2\bar{N} + 1$.

figure 4 it is easy to note that only in the case of interaction with vacuum, namely $N = 1$, do state remain entangled.

Let us stress that figures 1 and 3 (similarly figures 2 and 4) are plotted for the same initial values of the squeezing parameter, r, and the mean thermal photon number, $\bar{N}$, but for different models of interaction.

It is worth noting that in the case when a pure two-mode squeezed vacuum state interacts with a common reservoir initially in vacuum state ($\bar{N} = 0$) the resulting state, after an infinitely long interaction, is pure, but it is not vacuum. The final state is squeezed, and thus entangled.

This is the case because of the properties of the interaction Hamiltonian (1), which show up after non-local transformation of variables such as

$$
\begin{pmatrix} x_1 \\ p_1 \\ x_2 \\ p_2 \end{pmatrix} \rightarrow \begin{pmatrix} x_S \\ p_S \\ x_D \\ p_D \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ p_1 + p_2 \\ x_2 - x_1 \\ p_2 - p_1 \end{pmatrix}
\tag{25}
$$

where new variables are just sums and differences of the old ones.

Our initial state is pure and rewriting it in a new basis does not change its purity, while the interaction Hamiltonian (1) appears to couple to the reservoir only mode S described by sums, (x_S, p_S), leaving mode D described by differences, (x_D, p_D), unaffected [22]. Thus, only mode S interacts with the thermal reservoir and mode D undergoes free evolution. As a consequence, mode S first experiences decoherence and then dissipation to the state of the reservoir (see figures 5 and 6, decoherence for $\gamma t = 0$–0.5 and dissipation for $\gamma t > 0.5$). Therefore, if the vacuum is the environment of interest, mode S becomes vacuum after some time. Clearly both modes are pure, mode S because it is vacuum and mode D because it had evolved freely. Hence the final state is also pure, but it is still squeezed, because some squeezing 'survived' in mode D, and thus it is still entangled. If the reservoir initially is not in vacuum, the final state will be mixed. Here mode D enables us to span some kind of a decoherence-free subspace [23].

Of course in the case of interaction with two independent reservoirs, the interaction Hamiltonian (2) does not exhibit such a property under change of variables (25).

For the sake of clarity we plot the purity, $\text{Tr}[\rho^2]$, of the state of interest as a function of time for different initial values of the squeezing parameter, r, and the mean thermal photon
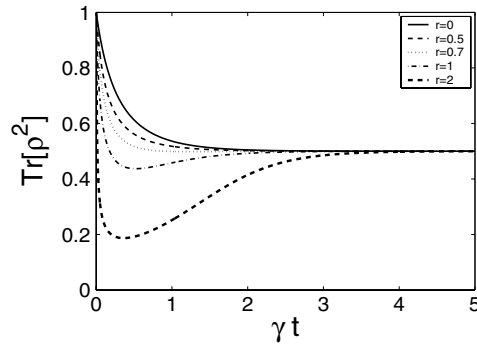
Figure 5. Purity, $\mathrm{Tr}[\rho^2]$, for the two-mode squeezed state as a function of γt, in the common reservoir model, for $\bar{N} = 0.5$ and different initial squeezing.
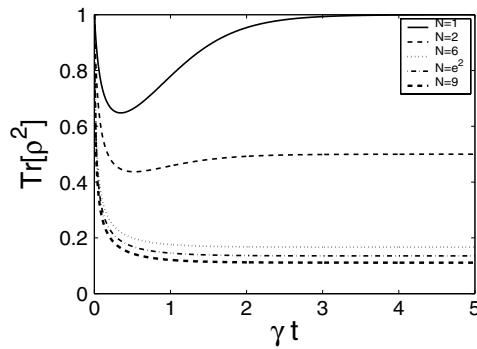

Figure 6. Purity, $\mathrm{Tr}[\rho^2]$, for the two-mode squeezed state as a function of γt, in the common reservoir model, for $r = 1$ and different mean thermal photon number, $N = 2\bar{N} + 1$.

number, $\bar{N}$ (see figures 5 and 6). For a Gaussian state described by the Wigner function with the variance matrix of the form (13) purity equals:

$$\mathrm{Tr}[\rho^2] = \frac{1}{\sqrt{\left(n_1^2 - c_1^2\right)\left(n_2^2 - c_2^2\right)}}. \tag{26}$$

4. Summary

Summarizing, for the coupling to the common reservoir, we noted that whether the state is entangled or not depends mostly on the initial degree of squeezing. If the state is initially sufficiently squeezed, i.e. according to condition (18), it will always remain entangled, independently of the strength of the coupling to the environment. Otherwise, the state initially entangled will become separable after the time given in equation (19). It is clearly a different result from that obtained for the model in which each of the two entangled modes interacts with its own reservoir. In such a case only the interaction with vacuum fluctuations does not lead to disentanglement [21]. Moreover, we have shown that in the case of interaction with a common reservoir being in a vacuum state, the state of interest, after infinitely long interaction, is not only still entangled, but is also pure.

Acknowledgments

References

[1] Barenco A, Deutsch D, Ekert A and Jozsa R 1995 *Phys. Rev. Lett.* **74** 4083
[2] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[3] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869
[4] Schumaker B L and Caves C M 1985 *Phys. Rev.* A **31** 3093
    Wu L A, Kimble H J, Hall J L and Wu H 1986 *Phys. Rev. Lett.* **57** 2520
    Barnett S M and Knight P L 1987 *J. Mod. Opt.* **34** 841
[5] Furusawa A *et al* 1998 *Science* **282** 706
[6] Jeong H, Lee J and Kim M S 2000 *Phys. Rev.* A **61** 052101
    Lee J, Kim M S and Jeong H 2000 *Phys. Rev.* A **62** 032305
[7] Kim M S and Lee J 2002 *Preprint* quant-ph/0203151
[8] Scheel S, Opatrný T and Welsch D-G 2000 *Preprint* quant-ph/0006026
    Scheel S and Welsch D-G 2001 *Phys. Rev.* A **64** 063811
[9] Hiroshima T 2001 *Phys. Rev.* A **63** 022305
[10] Paris M G A 2002 *J. Opt.* B **4** 442
[11] Bowen W P, Schnabel R, Lam P K and Ralph T C 2002 *Preprint* quant-ph/0209001
[12] Wigner E P 1932 *Phys. Rev.* **40** 749
[13] Simon R, Sudarshan E C G and Mukunda N 1987 *Phys. Rev.* A **36** 3868
    Simon R, Mukunda N and Dutta B 1994 *Phys. Rev.* A **49** 1567
[14] Peres A 1996 *Phys. Rev. Lett.* **77** 1413
[15] Horodecki M, Horodecki P and Horodecki R 1996 *Phys. Lett.* A **223** 1
    Horodecki P 1997 *Phys. Lett.* A **232** 333
[16] Simon R 2000 *Phys. Rev. Lett.* **84** 2726
[17] See Eisert J and Plenio M B 2003 *Preprint* quant-ph/0312071
    Giedke G, Duan L-M, Cirac J I and Zoller P 2001 *Quantum Inf. Comput* **1** 79
    Werner R F and Wolf M M 2001 *Phys. Rev. Lett.* **86** 3658
    Giedke G, Kraus B, Lewenstein M and Cirac J I 2001 *Phys. Rev. Lett.* **87** 167904
[18] Englert B G and Wódkiewicz K 2002 *Phys. Rev.* A **65** 054303
[19] Vidal G and Werner R F 2002 *Phys. Rev.* A **65** 032314
[20] Kim M S, de Oliveira F A M and Knight P L 1989 *Phys. Rev.* A **40** 2494
[21] Duan L M, Giedke G, Cirac J I and Zoller P 2000 *Phys. Rev. Lett.* **84** 2722
[22] Ekert A K 1991 Correlation in quantum optics *PhD Thesis* University of Oxford, OUALP-92-1 submitted
[23] Lidar D A and Whaley K B 2003 Decoherence-free subspaces and subsystems *Irreversible Quantum Dynamics* (*Springer Lecture Notes in Physics* vol 622) ed F Benatti and R Floreanini (Berlin: Springer) pp 83–120